

# Rate Splitting: An Idea Whose Time Has Come

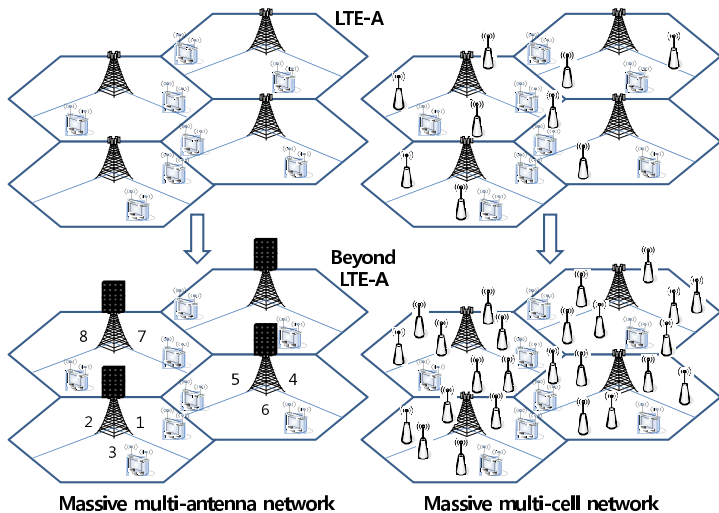
**Bruno Clerckx**

Dept. of Electrical and Electronic Engineering  
**Imperial College London**

Fifth Spanish Workshop on Signal Processing, Information Theory and  
Communications, Jan 2017

- ① Limitations of Current 4G and Emerging 5G Architecture
- ② The MISO Broadcast Channel and Partial CSIT
- ③ Fundamentals of Rate Splitting
- ④ Precoder Optimization
- ⑤ Extensions of Rate-Splitting
- ⑥ Conclusions and Future Challenges

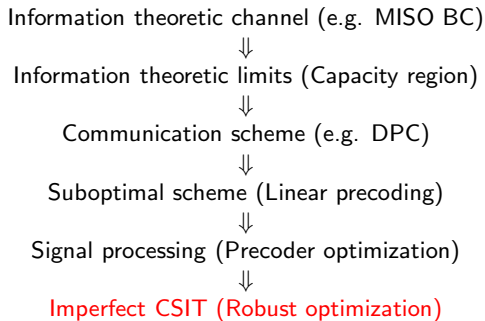
# MIMO Networks: Single-user, Multi-user, Multi-cell, Massive, Network, Cooperative, Coordinated,...



# Motivation for a New Physical Layer

- Big loss as the CSIT accuracy decreases.
- High CSIT accuracy has become increasingly difficult to satisfy
  - Dense HetNet, Massive MIMO
- So far, techniques designed for perfect CSIT applied to imperfect CSIT scenarios.
- Imperfect CSIT hardly avoidable.
- Wiser to design wireless networks from scratch accounting for imperfect CSIT?

# Motivation for a New Physical Layer



For example, robust optimization of  $\mathbf{p}_1, \dots, \mathbf{p}_K$  in

$$\mathbf{x} = \sum_{k=1}^K \mathbf{p}_k s_k.$$

BUT !!! The design is motivated by perfect CSIT to start with.

# A Bottom-up Approach

Information theoretic channel (e.g. MISO BC with Imperfect CSIT)



Information theoretic limits (Capacity region - unknown)



Alternative information theoretic limits (DoF region)



Communication scheme (Based on Rate-Splitting)



Suboptimal scheme (Linear precoding)



Signal processing (Precoder optimization)

For example, optimizing  $\mathbf{p}_c, \mathbf{p}_1, \dots, \mathbf{p}_K$  in

$$\mathbf{x} = \mathbf{p}_c s_c + \sum_{k=1}^K \mathbf{p}_k s_k$$

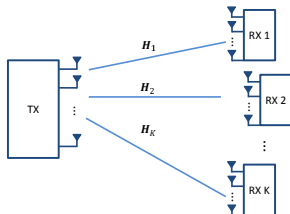
where  $\mathbf{p}_c s_c$  comes from Rate-Splitting.

Motivated by optimality in a DoF sense (multiplexing gain)

# The MISO Broadcast Channel and Partial CSIT

- 1 Limitations of Current 4G and Emerging 5G Architecture
- 2 The MISO Broadcast Channel and Partial CSIT
  - System model
  - Perfect CSIT
  - Imperfect CSIT
- 3 Fundamentals of Rate Splitting
- 4 Precoder Optimization
- 5 Extensions of Rate-Splitting
- 6 Conclusions and Future Challenges

# System model



$$y_k(t) = \mathbf{h}_k^H(t)\mathbf{x}(t) + n_k(t)$$

- $M$  transmit antennas and  $K$  single-antenna users ( $M \geq K$ ).
- Channel state (matrix):  $\mathbf{H}(t) = [\mathbf{h}_1(t), \dots, \mathbf{h}_K(t)]$ .
- In each  $t$ , transmitter obtains the estimate  $\hat{\mathbf{H}}(t)$  (i.e. CSIT).

# System model: Transmission and Linear precoding

## Linear precoding signal model:

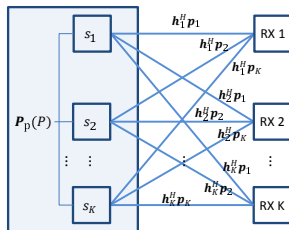
- Independent symbol streams:  $W_1, \dots, W_K \mapsto s_1, \dots, s_K$ .
- $t$  is dropped for simplicity.
- Unity average power:  $\mathbb{E}\{s_i s_k^*\} = 1$  if  $i = k$ , and 0 if  $i \neq k$ .
- Linear Precoding:

$$\mathbf{x} = \mathbf{p}_1 s_1 + \dots + \mathbf{p}_K s_K.$$

- Average power constraint:  $\sum_{k=1}^K \|\mathbf{p}_k\|^2 \leq P$ .
- $\mathbf{P}_p = [\mathbf{p}_1, \dots, \mathbf{p}_K]$  can be adapted based on CSIT

$$\mathbf{P}_p(\hat{\mathbf{H}}(1)), \mathbf{P}_p(\hat{\mathbf{H}}(2)), \dots, \mathbf{P}_p(\hat{\mathbf{H}}(T)).$$

# System model: SINR and Rate

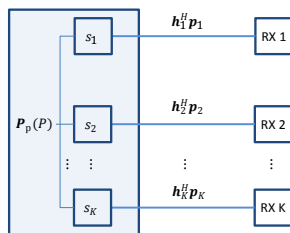


$$y_k = \underbrace{\mathbf{h}_k^H \mathbf{p}_k s_k}_{\text{desired signal}} + \underbrace{\mathbf{h}_k^H \sum_{i \neq k} \mathbf{p}_i s_i}_{\text{interference}} + \underbrace{n_k}_{\text{noise}}$$

- SINR (instantaneous):  $\text{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{p}_k|^2}{\sum_{i \neq k} |\mathbf{h}_k^H \mathbf{p}_i|^2 + \sigma_n^2}$ .
- Rate (instantaneous):  $R_k = \log_2(1 + \text{SINR}_k)$ .
- Ergodic Rate (for  $T \gg 1$ ):  $\mathbb{E}\{R_k\}$ .

# Perfect CSIT

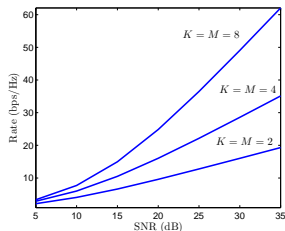
- Perfect CSIT:  $\hat{\mathbf{H}} = \mathbf{H}$ .
- Zero-Forcing (ZF) precoding:
  - $\mathbf{P}_p = \mathbf{H}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{B}$  where  $\mathbf{B}$  is diagonal.
  - This yields:  $\mathbf{p}_k \in \text{null}([\mathbf{h}_1, \dots, \mathbf{h}_{k-1}, \mathbf{h}_{k+1}, \dots, \mathbf{h}_K]^H)$ .



$$y_k = \mathbf{h}_k^H \mathbf{p}_k s_k + n_k$$

- Each user receives an interference-free stream.
- In other words, each user gets one full DoF.

# Perfect CSIT: Degrees of Freedom (DoF)



- DoF: fraction of an interference-free stream's capacity as  $P \rightarrow \infty$ .
- Considering the Ergodic rate:

$$d_k = \lim_{P \rightarrow \infty} \frac{\mathbb{E}\{R_k\}}{\log_2(P)}.$$

- For MISO, we have  $d_k \leq 1$  due to single-antenna receivers.
- Under perfect CSIT, ZF and equal power allocation achieves full DoF:

$$\sum_{k=1}^K d_k = K.$$

# Imperfect CSIT

What happens when CSIT is imperfect?

**Imperfect CSIT model:**

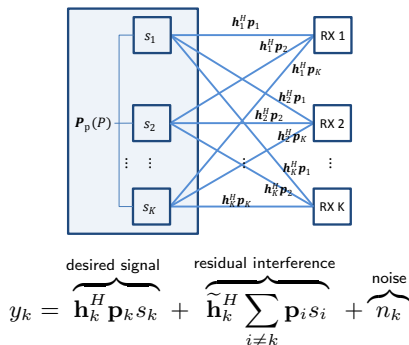
$$\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}$$
$$\mathbf{h}_k = \underbrace{\hat{\mathbf{h}}_k}_{\text{estimate}} + \underbrace{\tilde{\mathbf{h}}_k}_{\text{error}}$$

Estimate obtained through feedback or UL training [2].

- CSIT error power:  $\mathbb{E} \left\{ \|\tilde{\mathbf{h}}_k\|^2 \right\} = \sigma_{e,k}^2$ .
- CSIT error scaling:  $\alpha_k = \lim_{P \rightarrow \infty} -\frac{\log(\sigma_{e,k}^2)}{\log(P)}$
- It follows that:  $\mathbb{E} \left\{ \|\tilde{\mathbf{h}}_k\|^2 \right\} \sim P^{-\alpha_k}$ .
- Assume:  $\alpha_1, \dots, \alpha_K = \alpha$ .
  - $\alpha > 0$ : CSIT improves with  $P$  (e.g. increasing number of feedback bit).
  - $\alpha = 0$ : CSIT fixed with  $P$  (e.g. fixed number of feedback bit).
  - $\alpha = 1$ : CSIT perfect in a DoF sense (as we see next).

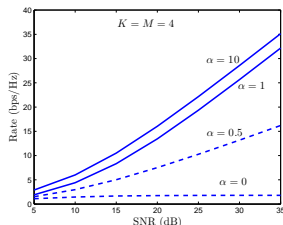
# Imperfect CSIT: Zero-Forcing

- ZF over the imperfect channel estimate:
  - $\mathbf{P}_p = \widehat{\mathbf{H}}(\widehat{\mathbf{H}}^H \widehat{\mathbf{H}})^{-1} \mathbf{B}$ .
  - This yields:  $\mathbf{p}_k \in \text{null} \left( \left[ \widehat{\mathbf{h}}_1, \dots, \widehat{\mathbf{h}}_{k-1}, \widehat{\mathbf{h}}_{k+1}, \dots, \widehat{\mathbf{h}}_K \right]^H \right)$ .



- Each user cannot enjoy an interference-free stream anymore.
- What happens to the DoF?

# Imperfect CSIT: DoF loss



- ZF and equal power allocation:  $\|\mathbf{p}_1\|^2 = \dots = \|\mathbf{p}_K\|^2 = \frac{P}{K}$ .

$$y_k = \underbrace{\mathbf{h}_k^H \mathbf{p}_k s_k}_{\text{desired signal} \sim P} + \underbrace{\tilde{\mathbf{h}}_k^H \sum_{i \neq k} \mathbf{p}_i s_i}_{\text{residual interference} \sim P^{1-\alpha}} + \underbrace{n_k}_{\text{noise} \sim P^0}$$

- Assume  $\alpha \in [0, 1]$ .
- $\text{SINR}_k \sim P^\alpha$  from which  $\mathbb{E}\{R_k\} = \log_2(P^\alpha) + O(1)$ .
- $d_k = \alpha$  from which the sum DoF [1, 2]:

$$\sum_{k=1}^K d_k = K\alpha.$$

# Imperfect CSIT: Interference

## Perfect CSIT:

- Inter-user interference can be fully eliminated.
- Full DoF is achieved.

## Partial CSIT with $\alpha \geq 1$ :

- Inter-user interference can be reduced to the level of noise.
- No DoF loss.

## Partial CSIT with $\alpha < 1$ :

- Inter-user interference cannot be reduced to the level of noise.
- Treating interference as noise causes DoF loss.

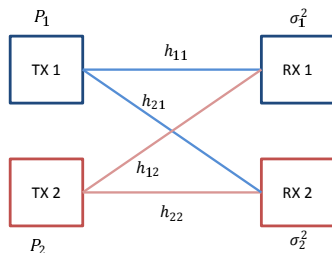
**If interference cannot be eliminated or reduced to noise level, why not decode it and remove it from the received signal (fully or in part)?**

Let us first take a step back, and look at the 2-user Interference Channel (IC).

# Fundamentals of Rate Splitting

- 1 Limitations of Current 4G and Emerging 5G Architecture
- 2 The MISO Broadcast Channel and Partial CSIT
- 3 Fundamentals of Rate Splitting**
  - Two-user Interference Channel
  - The MISO-BC with imperfect CSIT revisited
  - Sum-Rate enhancement and Feedback reduction
- 4 Precoder Optimization
- 5 Extensions of Rate-Splitting
- 6 Conclusions and Future Challenges

# Two-User Interference Channel (IC)



$$y_k = h_{k1}x_1 + h_{k2}x_2 + n_k$$

- Message  $W_k$  from TX- $k$  to RX- $k$ .
- Encoding:  $W_k \mapsto x_k$ .
- Decoding:  $y_k \mapsto \widehat{W}_k$ .

## Symmetric setup:

- $|h_{11}|^2 = |h_{22}|^2 = |h_d|^2$  and  $|h_{12}|^2 = |h_{21}|^2 = |h_c|^2$
- $P_1 = P_2 = P$  and  $\sigma_1^2 = \sigma_2^2 = \sigma^2$

# Two-User IC: Rate-Splitting

**Weak interference**  $|h_c|^2 < |h_d|^2$  (or general case):

- Not strong enough to **decode**, or weak enough to **treat as noise**.
- **Rate-Splitting**: part **decoded** by other and part **treated as noise**.
  - Split messages:  $W_k \mapsto W_{k0}, W_{k1} \mapsto x_{k0}, x_{k1}$ .
  - Split power:  $P_k \mapsto P_{k0}, P_{k1}$ .
  - RX-1 decodes  $x_{20}$  and  $x_1$  (composed of  $x_{10}, x_{11}$ ).
  - RX-2 decodes  $x_{10}$  and  $x_2$  (composed of  $x_{20}, x_{21}$ ).
- Reduces to **treat as noise** when  $P_{10} = P_{20} = 0$ .
  - i.e.  $|W_{10}| = |W_{20}| = 0$ .
  - $W_k \mapsto x_{k1}$ .
- Reduces to **decode** interference when  $P_{11} = P_{21} = 0$ .
  - i.e.  $|W_{11}| = |W_{21}| = 0$ .
  - $W_k \mapsto x_{k0}$ .
- **Bridges** the two in general [3].

# The MISO-BC with imperfect CSIT revisited

## Rate-Splitting for MISO-BC[4]:

- The general idea is to split messages.
- One part decoded by all, while the other treated as noise.

## But!

- In what proportion are messages split?
- How much power to allocate?
- How to transmit each part?

## Strategy:

- Private messages:
  - Parts which are treated as noise.
  - Received at the level of noise
- Common message(s):
  - Parts which are decoded by all.
  - Transmitted in a public manner.

# MISO-BC: Parts to treat as noise (private messages)

## Interference reduction through power control:

- Reduce allocated power to  $P^\alpha$ .
- Note that  $P^\alpha \leq P$  for  $\alpha \in [0, 1]$ .
- Equal power allocation:  $\|\mathbf{p}_1\|^2 = \dots = \|\mathbf{p}_K\|^2 = \frac{P^\alpha}{K}$ .

$$y_k = \underbrace{\mathbf{h}_k^H \mathbf{p}_k s_k}_{\text{desired signal } \sim P^\alpha} + \underbrace{\tilde{\mathbf{h}}_k^H \sum_{i \neq k} \mathbf{p}_i s_i}_{\text{residual interference } \sim P^{\alpha-\alpha} = P^0} + \underbrace{n_k}_{\text{noise } \sim P^0}$$

- Interference is reduced to noise level  $\sim P^0$ .
- This also limits desired power  $\sim P^\alpha$ .
- DoF is maintained:  $d_k = \alpha$  and  $\sum_{k=1}^K d_k = K\alpha$ .
- Only power levels (scalings) from 0 to  $\alpha$  are occupied.
- The remaining power levels ( $\alpha$  to 1) are freed for the other parts.

# MISO-BC: Parts to decode (common message)

Superpose  $W_c \mapsto s_c$  (with precoder  $\mathbf{p}_c$ ) to be decoded by all users.

$$\mathbf{x} = \mathbf{p}_c s_c + \sum_{k=1}^K \mathbf{p}_k s_k$$

where  $\|\mathbf{p}_c\|^2 = P - P^\alpha \sim P$  and  $\|\mathbf{p}_1\|^2 = \dots = \|\mathbf{p}_K\|^2 = \frac{P^\alpha}{K} \sim P^\alpha$

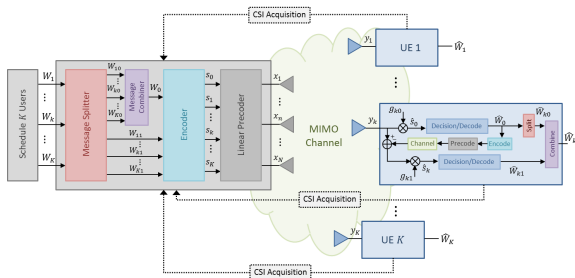
$$y_k = \underbrace{\mathbf{h}_k^H \mathbf{p}_c s_c}_{\sim P} + \underbrace{\mathbf{h}_k^H \mathbf{p}_k s_k}_{\sim P^\alpha} + \underbrace{\tilde{\mathbf{h}}_k^H \sum_{i \neq k} \mathbf{p}_i s_i}_{\sim P^0} + \underbrace{n_k}_{\sim P^0}$$

- $\text{SINR}_{c,k} \sim P^{1-\alpha}$  from which  $\mathbb{E}\{R_{c,k}\} = \log_2(P^{1-\alpha}) + O(1)$ .
- DoF of common message:  $d_c = 1 - \alpha$ .
- SIC is used to remove  $s_c$ , as it is decoded by all.
- DoF of private messages is maintained:  $d_k = \alpha$ .
- Sum DoF is boosted:  $d_c + \sum_{k=1}^K d_k = (1 - \alpha) + K\alpha$ .

What remains is to load both parts (private and common) with user data.

# MISO-BC: Rate-Splitting

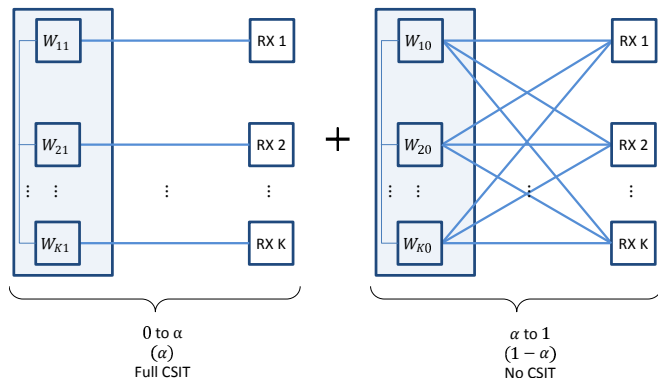
Instead of a new common message,  $s_c$  is loaded with part of user messages.



- Split message of user-1 :  $W_1 \mapsto W_{10}, W_{11}$ .
- Common part:  $W_{10} \mapsto s_c$ , decoded by all users but intended to users-1.
- Private part:  $W_{11} \mapsto s_1$  decoded by user-1.
- $W_2, \dots, W_K \mapsto s_2, \dots, s_K$  decoded by corresponding users.

**Splitting can be done for other (or all) users as in figure [24].**

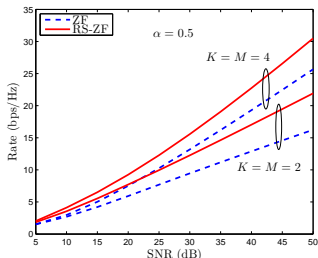
# MISO-BC: Weighted sum interpretation



Decomposed into a weighted superposition of two networks

- Perfect CSIT.
  - Achieves sum DoF of  $K$ .
  - Weighted by  $\alpha$ .
- No CSIT
  - Achieves sum DoF of 1.
  - Weighted by  $1 - \alpha$ .

# MISO-BC: DoF with RS



## Proposition

In the  $K$  user MISO-BC with partial CSIT, sum DoF achieved by ZF is given by

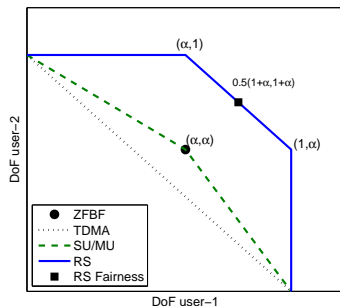
$$d_{\Sigma}^{\text{ZF}} = K\alpha$$

while the sum DoF achieved by RS-ZF is given by

$$d_{\Sigma}^{\text{RS}} = 1 + (K - 1)\alpha.$$

Optimality of the RS DoF shown in [5].

# MISO-BC: Two-User DoF region

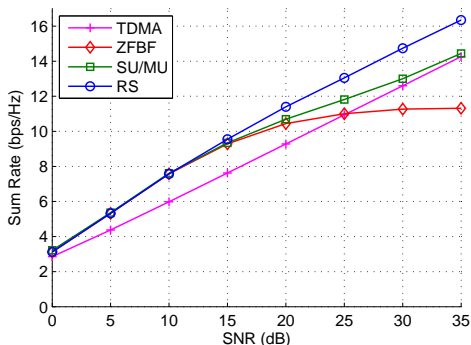


- Assume splitting for user-1
  - user-1 DoF:  $d_c + d_1 = (1 - \alpha) + \alpha = 1$ .
  - user-2 DoF:  $d_2 = \alpha$ .
- Time-sharing between splitting for user-1 and user-2.
- Compared to time-sharing between ZF and TDMA.

# Sum-Rate enhancement and Feedback reduction

## From DoF to rate analysis:

- So far we have looked at the DoF gains of RS ( $P \rightarrow \infty$ ).
- Sum-rate enhancement (slope gain and/or SNR gain) over ZF, TDMA, switching between TDMA/ZF (SU/MU)[6].
- $M = 4$  antennas,  $K = 2$  users, and  $B = 15$  bits.



# Precoder Optimization

- 1 Limitations of Current 4G and Emerging 5G Architecture
- 2 The MISO Broadcast Channel and Partial CSIT
- 3 Fundamentals of Rate Splitting
- 4 Precoder Optimization**
  - Ergodic Sum-Rate Maximization
  - Robust Max-Min Fairness
- 5 Extensions of Rate-Splitting
- 6 Conclusions and Future Challenges

# Precoder Optimization

Recall that the RS (linearly precoded) signal model is:

$$\mathbf{x} = \mathbf{p}_c s_c + \sum_{k=1}^K \mathbf{p}_k s_k$$

- Precoding matrix:  $\mathbf{P} = [\mathbf{p}_c, \mathbf{p}_1, \dots, \mathbf{p}_K]$ .
- Power constraint:  $\text{tr}(\mathbf{P}\mathbf{P}^H) \leq P$ .
- So far we considered simple barely optimized designs (ZF, random).
- The choice of  $\mathbf{P}$  influences  $R_c, R_1, \dots, R_K$ .

## Challenges

- Transmitter only knows  $\hat{\mathbf{H}}$  and not  $\mathbf{H}$ .
- Instantaneous  $R_c, R_1, \dots, R_K$  not known by the transmitter.
- Transmission should be carried out at reliable (decodable) rates.

# Ergodic Sum-Rate Maximization

RS problem [11]:

$$\mathcal{R}_{\text{RS}}(P) : \begin{cases} \max_{\bar{R}_c, \mathbf{P}} & \bar{R}_c + \sum_{k=1}^K \bar{R}_k \\ \text{s.t.} & \bar{R}_{c,k} \geq \bar{R}_c, \forall k \in \mathcal{K} \\ & \text{tr}(\mathbf{P}\mathbf{P}^H) \leq P \end{cases}$$

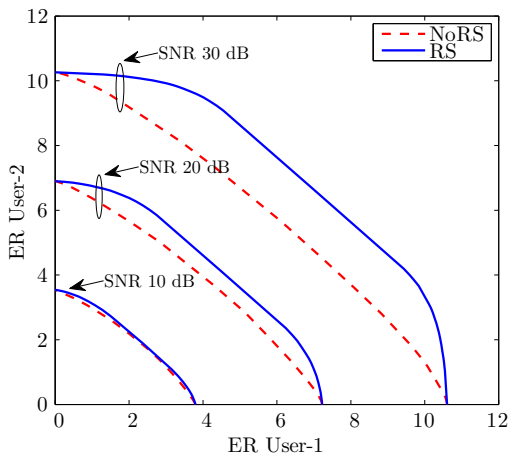
as opposed to the conventional (NoRS) formulation

$$\mathcal{R}(P) : \begin{cases} \max_{\mathbf{P}_p} & \sum_{k=1}^K \bar{R}_k \\ \text{s.t.} & \text{tr}(\mathbf{P}_p \mathbf{P}_p^H) \leq P. \end{cases}$$

- Stochastic optimization problem (due to expectations inside the ARs).
- Even a deterministic version is non-convex and very difficult.
- WMMSE approach can efficiently handle sum rate problems.

# Ergodic Sum-Rate Maximization: Two-user ER region

- More generally, we can solve the Weighted ESR problem.



- Shows the ER trade-offs between the two users.

# Robust Max-Min Fairness

Non-Ergodic transmission over  $T = 1$  random state  $\{\mathbf{H}, \widehat{\mathbf{H}}\}$ .

- For  $k$ th user, CSIT errors bounded by sphere with radius  $\delta_k$ :

$$\mathbb{H}_k = \left\{ \mathbf{h}_k \mid \mathbf{h}_k = \widehat{\mathbf{h}}_k + \widetilde{\mathbf{h}}_k, \|\widetilde{\mathbf{h}}_k\| \leq \delta_k \right\}$$

- For any  $\mathbf{P}$ , worst-case rates defined as:

$$\bar{R}_{c,k} = \min_{\mathbf{h}_k \in \mathbb{H}_k} R_{c,k}(\mathbf{h}_k) \quad \text{and} \quad \bar{R}_k = \min_{\mathbf{h}_k \in \mathbb{H}_k} R_k(\mathbf{h}_k).$$

- For given  $\widehat{\mathbf{H}}$ , transmission at worst-case rates is reliable (robust).

**Rate-Splitting revisited [12]:** Sharing the common message

- $W_k \mapsto W_{k0}, W_{k1}$  for all  $k \in \{1, \dots, K\}$ .
- $W_{10}, \dots, W_{K0} \mapsto s_c$ .
- $W_{11}, \dots, W_{K1} \mapsto s_1, \dots, s_K$ .

# Robust Max-Min Fairness

$$\mathcal{R}_{\text{RS}}(P) : \begin{cases} \max_{\bar{\mathbf{c}}, \mathbf{P}} & \min_{k \in \mathcal{K}} (\bar{R}_k + \bar{C}_k) \\ \text{s.t.} & \bar{R}_{\mathbf{c}, k} \geq \sum_{i=1}^K \bar{C}_i, \forall k \in \mathcal{K} \\ & \bar{C}_k \geq 0, \forall k \in \mathcal{K} \\ & \text{tr}(\mathbf{P}\mathbf{P}^H) \leq P. \end{cases}$$

where  $\bar{\mathbf{c}} = [\bar{C}_1, \dots, \bar{C}_M]$ .

- Portion of the common message rate given to user  $k$ :  $\bar{C}_k$ .
- Sum of all portions:  $\sum_{k=1}^K \bar{C}_k = \bar{R}_{\mathbf{c}} = \min_i \bar{R}_{\mathbf{c}, i}$ .
- Rate of user  $k$ :  $\bar{R}_k + \bar{C}_k$  (private and common portions).

Classical (NoRS) problem formulated as:

$$\mathcal{R}(P) : \begin{cases} \max_{\mathbf{P}_p} & \min_{k \in \mathcal{K}} \bar{R}_k \\ \text{s.t.} & \text{tr}(\mathbf{P}_p \mathbf{P}_p^H) \leq P. \end{cases}$$

# Robust Max-Min Fairness: Simulation results

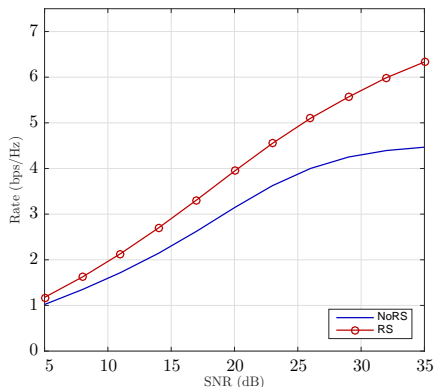


Figure:  $K = M = 3$  and  $\delta_1, \delta_2, \delta_3 = 0.1$ .

- NoRS saturates due to non-scaling CSIT errors.
- RS avoids saturation and performs better across all SNRs.

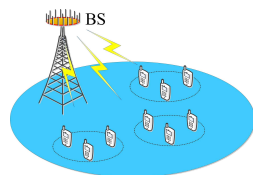
# Extensions of Rate-Splitting

- 1 Limitations of Current 4G and Emerging 5G Architecture
- 2 The MISO Broadcast Channel and Partial CSIT
- 3 Fundamentals of Rate Splitting
- 4 Precoder Optimization
- 5 Extensions of Rate-Splitting**
  - Massive MISO
  - Multi-Cell Coordination
  - Overloaded systems
  - Multigroup multicast beamforming
  - Multiuser Millimeter Wave Beamforming
  - RF Impairments
- 6 Conclusions and Future Challenges

**Massive MIMO challenge:** the huge demand for accurate CSIT.

**The use of Rate-Splitting:**

- The constraint:  $R_c = \min_k \{R_{c,k}\}$ .
- This highly reduces the gain when  $K$  is large.
- Channel statistics  $\mathbf{R}_k$  can be further exploited.
- Large training and feedback overhead.



**User grouping based on spatial correlation:**

- Two-tier precoding [14, 15, 16]

$$\mathbf{x} = \sqrt{\frac{P}{K}} \sum_{g=1}^G \mathbf{B}_g \mathbf{W}_g \mathbf{s}_g,$$

- Users in  $g$ -th group share the same channel statistics:  $\mathbf{R}_g$ .
- $\mathbf{B}_g$ : outer-precoding matrix based on channel statistics.
- $\mathbf{W}_g$ : inner-precoding matrix designed based on short-term effective channel estimates:  $\hat{\mathbf{H}}_g = \mathbf{B}_g^H \hat{\mathbf{H}}_g$ .

# Massive MISO: Hierarchical Rate-Splitting (HRS)

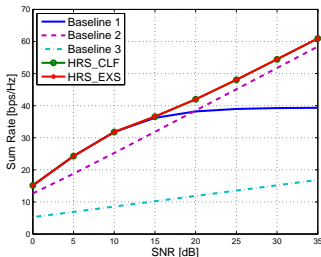
- Overlap between the eigen-subspaces  $\Rightarrow$  inter-group interference.
- Imperfect CSIT  $\Rightarrow$  intra-group interference.
- **Hierarchical Rate-Splitting**[10]: a hierarchy of common messages to combat the inter-group and intra-group interference in Massive MIMO

$$\mathbf{x} = \overbrace{\sqrt{P_{sc}} \mathbf{w}_{sc} s_{sc}}^{\text{system common msg.}} + \sum_{g=1}^G \mathbf{B}_g \left( \overbrace{\sqrt{P_{cg}} \mathbf{w}_{cg} s_{cg}}^{\text{group common msg.}} + \overbrace{\sqrt{P_{gk}} \mathbf{W}_g \mathbf{s}_g}^{\text{private msgs.}} \right)$$

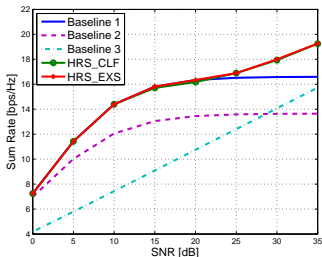
- System common msg. decoded by all users: for inter-group interference.
- Group common msg. decoded by group: for intra-group interference

# Massive MISO: Simulation results

- HRS under imperfect CSIT,  $M = 100$ ,  $K = 12$ ,  $\tau^2 = 0.4$



(a) disjoint eigen-subspace

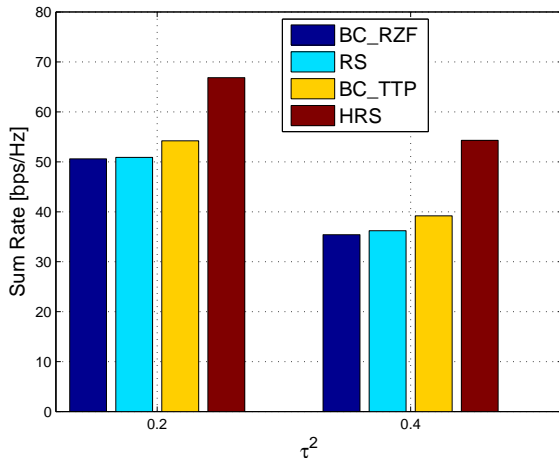


(b) overlapping eigen-subspace

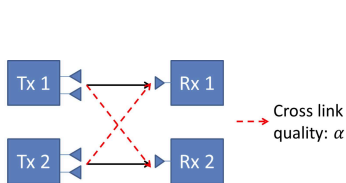
- HRS behaves as two-tier BC at low to medium SNR.
- HRS achieves a non-saturating sum rate.
- HRS decreases the complexity of precoder design and scheduling.
- HRS increases the complexity of the encoders and decoders.

# Massive MISO: Simulation results

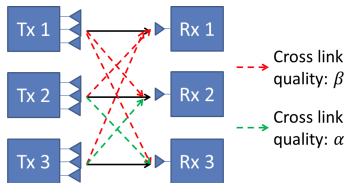
- $M = 100$ ,  $K = 12$ ,  $\tau^2 = 0.4$ ,  $SNR = 30dB$ , disjoint eigen-subspaces



# Multi-Cell Coordination: Topological Rate-Splitting (TRS)



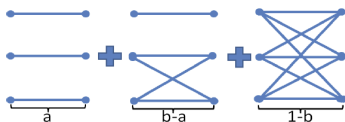
(c) two-cell scenario [17]



(d) three-cell scenario [18]

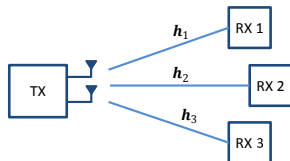
|     | Tx1 | Tx2 | Tx3 |
|-----|-----|-----|-----|
| Rx1 |     | b   | b   |
| Rx2 | b   |     | a   |
| Rx3 | b   | a   |     |

(e) CSIT pattern



(f) Weighted-sum interpretation [18]

# Overloaded systems



- Overloaded scenarios:  $K > M$ .
- Scheduling over orthogonal resource blocks (time/frequency).
- Serve at most  $M$  users at a time.
- Reduces to conventional MISO BC in each block.
- With perfect CSIT, achieves DoF  $M$  in each block.

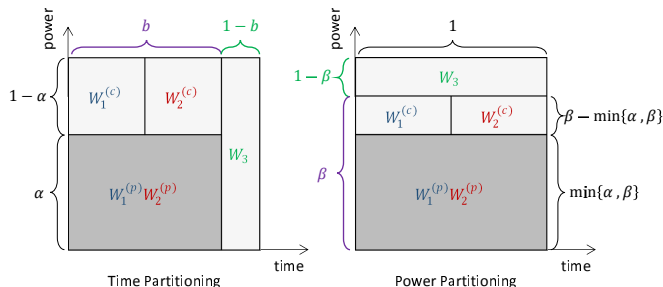
Consider a scenario where some user have little or no CSIT:

- IoT with many devices.
- Low-power sensor-like receivers.
- Can be served using the common message in the RS scheme [19].

# Overloaded systems: Three-User example

- System:  $M = 2$  antennas and  $K = 3$  users.
- CSIT:  $\alpha_1 = \alpha_2 = \alpha$  and  $\alpha_3 = 0$ .

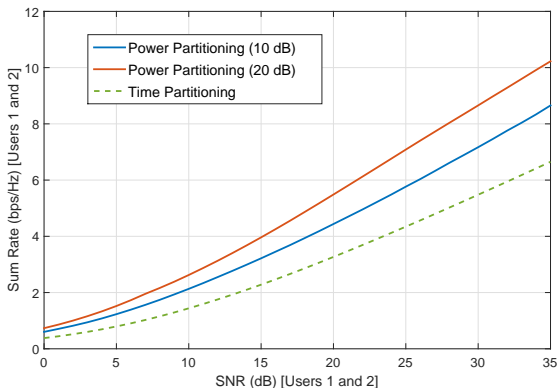
## Scheduling approach



## Power partitioning

- A **superposition of non-orthogonal layers and an orthogonal layer**
- Power partitioning achieves the optimum DoF region

# Overloaded systems: Numerical Results



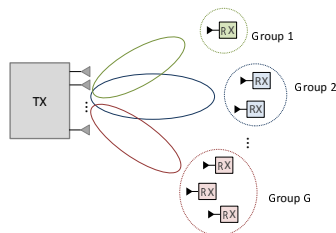
**Figure:** Sum rate of user-1 and user-2 while maintaining the same rate for user-3 for the cases when the long-term SNR of user-3 is 10 dB and 20 dB lower than user-1,2. The parameters taken are  $\alpha = 0.5$  and  $b = 0.5$ .

# Multigroup multicast beamforming

Users clustered into groups depending on content demand.

- $K$  users grouped into  $\mathcal{G}_1, \dots, \mathcal{G}_G$ .
- One message for each group:  $W_1, \dots, W_G$ .
- Classical beamforming:

$$\mathbf{x} = \sum_{g=1}^G \mathbf{p}_g s_g.$$



**Achieving max-min fairness (perfect CSIT):**

$$\mathcal{R}(P) : \begin{cases} \max_{\mathbf{P}_P} & \min_{g \in \{1, \dots, G\}} \min_{i \in \mathcal{G}_g} R_i \\ \text{s.t.} & \sum_{g=1}^G \|\mathbf{p}_g\|^2 \leq P. \end{cases}$$

- **Overloaded scenarios:**  $M$  is not enough for interference nulling.
- Rate saturation (even with perfect CSIT) due to inter-group interference.

# Multigroup multicast beamforming: Simulation results

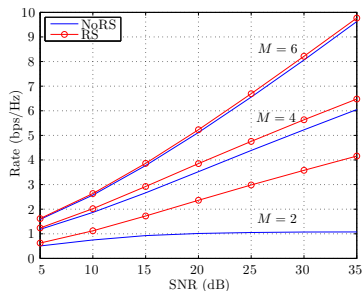


Figure:  $K = 6$  users,  $G = 3$  groups,  $|\mathcal{G}_1| = 1$ ,  $|\mathcal{G}_2| = 2$  and  $|\mathcal{G}_3| = 3$ .

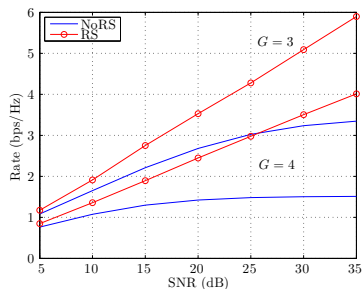
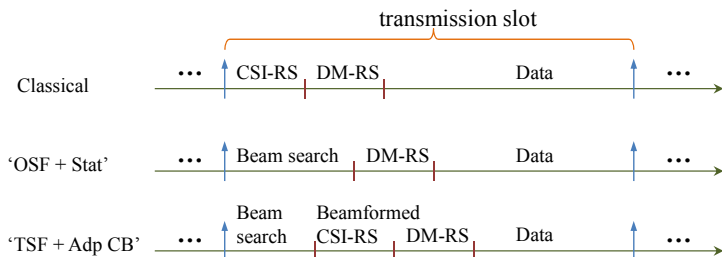


Figure:  $M = 4$  antennas,  $|\mathcal{G}_g| = 2$  users per group,  $G = 3$  and 4.

RS to mitigate inter-group interference in overloaded scenarios [13].

# Multiuser Millimeter Wave Beamforming

Signalling and feedback procedure [25]



# Multiuser Millimeter Wave Beamforming

RS to save second-stage channel training and feedback [25].

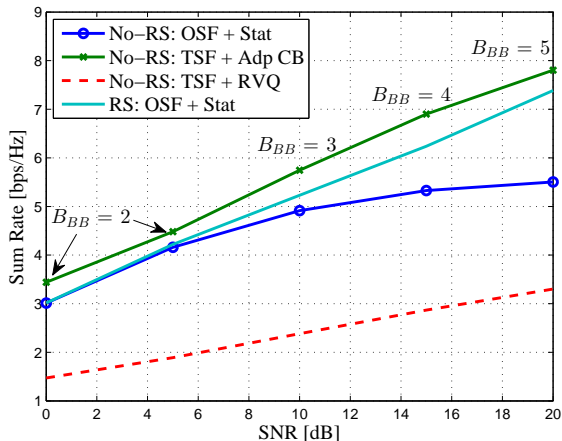
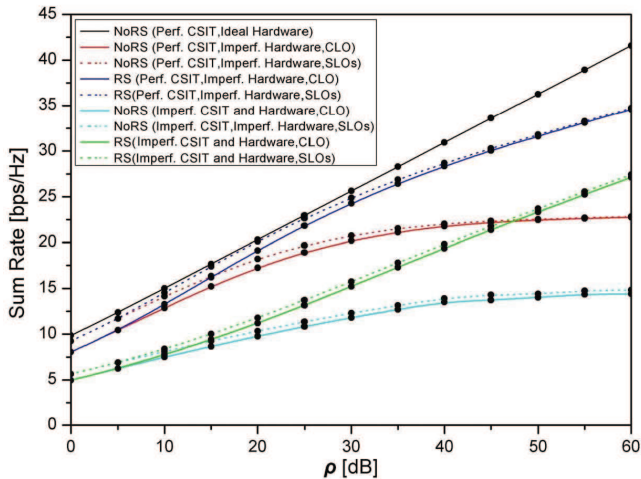


Figure: No-RS with extra feedback versus RS.  $M = 32$ ,  $K = 4$ ,  $B_{RF} = 4$ .

# RF Impairments

RS to mitigate phase noise impairments [26].



# Conclusions and Future Challenges

- ① Limitations of Current 4G and Emerging 5G Architecture
- ② The MISO Broadcast Channel and Partial CSIT
- ③ Fundamentals of Rate Splitting
- ④ Precoder Optimization
- ⑤ Extensions of Rate-Splitting
- ⑥ Conclusions and Future Challenges**

# Conclusions and Future Challenges

- 4G and current 5G candidates rely on private message transmissions
  - Such a strategy is only motivated in the presence of perfect CSIT
  - Apply techniques designed for perfect CSIT to imperfect CSIT
- RS relies on superposed transmission of common and private messages
  - Well known for SISO IC but actually useful for a wide range of scenarios.
  - Motivated by information theory for the realistic scenario of imperfect CSIT.
  - Benefits: spectral and energy efficiencies, reliability, CSI feedback overhead reduction.
- RS through the transmission of common and private message would have fundamental changes in the design of PHY and Lower MAC.
  - A gold mine of research problems for academia and of standard specification issues for industry
- The standardization of rate-splitting can leverage 3GPP current study/work items

# Future Challenges: A gold mine of research problems

## Introduction

- Overview, open problems, impact on standard specifications and operational challenges [24].

## Fundamental Limits

- DoF region for K-user MISO BC with imperfect CSIT [5].
- Capacity region of K-user MISO BC with imperfect CSIT: DPC + RS?
- DoF region for MIMO BC with imperfect CSIT [7, 8, 9].
- DoF region of overloaded MISO BC with imperfect CSIT [19].
- DoF region for MISO IC with imperfect CSIT [18]. TRS?
- DoF region for MIMO IC with imperfect CSIT [8]. RS + IA?
- Interplay between RS and coded caching.

## Optimization

- Ergodic sum-rate maximization for BC [11].
- Robust Max-Min Fairness for BC [12].
- RS beamforming optimization for other types of channels.

## PHY challenges

- Finite SNR rate analysis [6].
- Energy efficiency of RS-based transmission.
- Space-time/frequency RS [22, 23, 6].
- RS with multi-carrier transmissions.

## PHY challenges (continued)

- RS with non-linear precoding.
- Diversity (and BER) performance of RS-based strategies.
- RS for Multigroup Multicast [13].
- RS/HRS for Massive MIMO [10].
- RS as a way to combat pilot contamination.
- RS to mitigate hardware impairments.
- RS in higher frequency bands operation (e.g. millimeter-wave).
- RS-based network MIMO.
- Coordination/cooperation among distributed antennas in homogeneous and heterogeneous network deployments.
- RS in half-duplex relay.
- RS in full duplex.
- RS and NOMA/MUST [19].
- RS and superposition of multicast and unicast messages.
- RS and physical layer security.

## PHY/MAC challenges

- User pairing and scheduling of common and private messages.
- RS design with Quality of Experience (QoE) and traffic constraints.

# Future Challenges

## Performance Analysis

- Performance analysis of RS using stochastic geometry.

## Standardization

- Link and system-level evaluations of RS.
- MIMO receiver implementation.
- Transmission schemes/mode.
- CSI feedback mechanisms.
- Downlink and uplink signaling.

# References I

- [1] N. Jindal, "MIMO Broadcast Channels With Finite-Rate Feedback," *Trans. on Info Theory*, vol. 52, no. 11, pp. 5045-5060, Nov. 2006.
- [2] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, Multiuser MIMO achievable rates with downlink training and channel state feedback, *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2845-2866, Jun. 2010.
- [3] A. El Gamal and Y.-H. Kim, *Network information theory*, Cambridge university press, 2011.
- [4] S. Yang, M. Kobayashi, D. Gesbert, and X. Yi, Degrees of freedom of time correlated MISO broadcast channel with delayed CSIT, *IEEE Trans. Inf. Theory*, vol. 59, no. 1, pp. 315-328, Jan. 2013.
- [5] A.G. Davoodi, S.A. Jafar, "Aligned Image Sets under Channel Uncertainty: Settling a Conjecture by Lapidoth, Shamai and Wigger on the Collapse of Degrees of Freedom under Finite Precision CSIT," arXiv:1403.1541.
- [6] C. Hao, Y. Wu and B. Clerckx, "Rate analysis of two-receiver MISO broadcast channel with finite rate feedback: A rate-splitting approach," *IEEE Trans. on Comm.*, vol. 63, no. 9, pp. 3232-3246, Sept. 2015.
- [7] J. Chen and P. Elia, Symmetric two-user MIMO BC with evolving feedback, in *Inf. Theory and Applications Workshop (ITA)*, 2014, San Diego, CA, Feb 2014, pp. 15.
- [8] C. Hao, B. Rassouli and B. Clerckx, "Achievable DoF Regions of MIMO Networks with Imperfect CSIT," submitted to *IEEE Trans. on Info Theory*, arXiv:1603.07513.
- [9] B. Yuan, S.A. Jafar, "Elevated Multiplexing and Signal Space Partitioning in the 2 User MIMO IC with Partial CSIT," *IEEE SPAWC* 2016.
- [10] M. Dai, B. Clerckx, D. Gesbert and G. Caire, "A Rate Splitting Strategy for Massive MIMO with Imperfect CSIT," *IEEE Trans. on Wireless Comm.*, vol. 15, no. 7, pp. 4611-4624, July 2016.
- [11] H. Joudeh and B. Clerckx, "Sum-Rate Maximization for Linearly Precoded Downlink Multiuser MISO Systems with Partial CSIT: A Rate-Splitting Approach," *IEEE Trans. on Comm.*, *IEEE Trans. on Comm.* vol. 64, no. 11, pp. 4847-4861, Nov 2016.
- [12] H. Joudeh and B. Clerckx, "Robust Transmission in Downlink Multiuser MISO Systems: A Rate-Splitting Approach," *IEEE Trans. on Sig. Proc.* Vol. 64, No. 23, pp. 6227-6242, Dec 2016.
- [13] H. Joudeh and B. Clerckx, "A Rate-Splitting Strategy for Max-Min Fair Multigroup Multicasting," *IEEE SPAWC* 2016.
- [14] C. Lim, T. Yoo, B. Clerckx, B. Lee, and B. Shim, "Recent trend of multiuser MIMO in LTE-advanced," *IEEE Commun. Mag.*, vol. 51, no. 3, pp. 127-135, March 2013.
- [15] A. Adhikary, J. Nam, J. Ahn and G. Caire. "Joint spatial division and multiplexing - the large-scale array regime," *IEEE TIT*, Oct. 2013.

# References II

- [16] J. Chen and V. Lau, "Two-tier precoding for FDD multi-cell massive MIMO time-varying interference networks," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 12301238, June 2014.
- [17] X. Yi, D. Gesbert, S. Yang, and M. Kobayashi, On the dof of the multiple-antenna time correlated interference channel with delayed csit, in 2012 Conference Record of the Forty Sixth Asilomar Conference on Signals, Systems and Computers (ASILOMAR), Nov 2012, pp. 15661570.
- [18] C. Hao and B. Clerckx, "MISO Networks with Imperfect CSIT: A Topological Rate Splitting Approach," accepted to *IEEE Trans on comm.*, arXiv:1602.03768.
- [19] E. Piovano, H. Joudeh and B. Clerckx, "Overloaded MU-MISO Transmission with Imperfect CSIT," 2016 Asilomar Conference on Signals, Systems, and Computers.
- [20] J. Zhang, F. Engelmann and P. Elia, "Coded caching for reducing CSIT-feedback in wireless communications," Allerton, Monticello, IL, 2015, pp. 1099-1105.
- [21] B. Rassouli, C. Hao and B. Clerckx, "DoF Analysis of the MIMO Broadcast Channel with Alternating/Hybrid CSIT," *IEEE Trans. on Info Theory*, vol. 62, no. 3, pp. 1312-1325, Mar. 2016.
- [22] R. Tandon, S. Jafar, S. S. Shitz, and H. Poor, On the synergistic benefits of alternating CSIT for the MISO broadcast channel, *IEEE Trans. Inf. Theory*, vol. 59, no. 7, pp. 41064128, Jul. 2013.
- [23] J. Chen and P. Elia, Optimal DoF region of the two-user MISO-BC with general alternating CSIT, in *Proc. Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, Nov. 2013, pp. 18601864.
- [24] B. Clerckx, H. Joudeh, C. Hao, M. Dai and B. Rassouli, "Rate Splitting for MIMO Wireless Networks: A Promising PHY-Layer Strategy for LTE Evolution," *IEEE Comm. Mag*, pp 98-105, May 2016.
- [25] M. Dai and B. Clerckx, "Multiuser Millimeter Wave Beamforming Strategies with Quantized and Statistical CSIT," submitted to *IEEE Trans. on Wireless Comm.*
- [26] A. Papazafeiropoulos, B. Clerckx, and T. Ratnarajah, "Rate-Splitting to Mitigate Residual Transceiver Hardware Impairments in Massive MIMO Systems," submitted to *IEEE Trans. on Veh. Tech.*